

SOLVING AN UNBALANCED TRANSPORTATION PROBLEM WITH DODECAGONAL FUZZY NUMBERS

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ABSTRACT

In this paper, we propose a new approach for the solution of fuzzy unbalanced transportation problem under a fuzzy environment in which transportation costs are taken as fuzzy dodecagonal numbers. The numbers and fuzzy values are predominantly used in various fields such as experimental sciences, artificial intelligence, etc. because of their uncertainty. Here, we are converting fuzzy dodecagonal numbers into crisp value by using Robust Ranking technique and then solved by the proposed method for the transportation problem.

KEYWORDS

Transportation Problem, Dodecagonal Fuzzy Numbers, Robust Ranking, Crisp numbers, Defuzzification.

INTRODUCTION

The largest part of the literature is based on crisp transport problems, but in real circumstances the ambiguity in data requires the use of generalized fuzzy number and this is the main motivation behind this study. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. Transportation problem was originally introduced and developed by Hitchcock in 1941, in which the parameters like transportation cost, demand and supply are crisp values. But in the present world the transportation parameters may be uncertain due to many uncontrolled factors. So to deal the problems with imprecise information Zadeh [4] introduced the concept of fuzziness. Many authors discussed the solution of FTP with various fuzzy numbers. Chen [1] introduced the concept of generalized fuzzy numbers to deal problems with unusual membership function. Most of researchers applied generalized fuzzy numbers to solve the real life problems. Chandrasekaran, S, Kokila, G and Junu Saju [2] solved Ranking of Heptagon Number using Zero Suffix Method and Dr.A.Sahaya Sudha, S.Karunambigai [4] Solving a Transportation Problem using a Heptagonal Fuzzy Number. Edithstine Rani Mathew, Sunny Joseph Kalayathankal [5] "A New Ranking Method Using Dodecagonal Fuzzy Number to Solve Fuzzy Transportation Problem", Dr.T. Hemamalini, and M.Revathi[6] Solved "Ranking of Heptagonal fuzzy numbers" and Edithstine Rani Mathew, Sunny Joseph Kalayathankal[7] proposed A New Ranking Method Using Dodecagonal Fuzzy Number to Solve Fuzzy Transportation Problem.

In this paper is organized as follows, in section II, Some basic definition in section III, proposed algorithm followed by a Numerical example using Monalisha's approximation method and in section IV, finally the paper is concluded.

II. PRELIMINARIES

II.1. FUZZY SET [FS]:

A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit interval $[0, 1]$. (i.e.): $\mu_A : X \rightarrow [0, 1]$.

II.2. FUZZY NUMBER [FN]:

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1. The weight is called the membership function.

A fuzzy number \bar{A} is a convex normalized fuzzy set on the real line R such that

- There exist at least one $x \in R$ with $\mu_{\bar{A}}(x) = 1$.
- $\mu_{\bar{A}}(x)$ is piecewise continuous.

II.3.DODECAGONAL FUZZY NUMBER

The membership function of dodecagonal fuzzy number $\bar{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$.

Where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}$ are real numbers and is given by

$$\mu_{\bar{A}}(x) = \left\{ \begin{array}{ll} 0, & x \leq a_1 \\ k_1 \left(\frac{x - a_1}{a_2 - a_1} \right), & a_1 \leq x \leq a_2 \\ k_1, & a_2 \leq x \leq a_3 \\ k_1 + (1 - k_1) \left(\frac{x - a_3}{a_4 - a_3} \right), & a_3 \leq x \leq a_4 \\ k_2 a_4 \leq x \leq a_5 \\ k_2 + (1 - k_2) \left(\frac{x - a_5}{a_6 - a_5} \right), & a_5 \leq x \leq a_6 \\ 1, & a_6 \leq x \leq a_7 \\ k_2 + (1 - k_2) \left(\frac{a_8 - x}{a_8 - a_7} \right), & a_7 \leq x \leq a_8 \\ k_2, & a_8 \leq x \leq a_9 \\ k_1 + (1 - k_1) \left(\frac{a_{10} - x}{a_{10} - a_9} \right) a_9 \leq x \leq a_{10} \\ k_1 a_{10} \leq x \leq a_{11} \\ k_1 \left(\frac{a_{12} - x}{a_{12} - a_{11}} \right) a_{11} \leq x \leq a_{12} \\ 0 & a_{12} \leq x \end{array} \right.$$

III.1.PROCEDURE FOR SOLVING DODECAGONAL FUZZY NUMBER USING MONALISHA APPROXIMATION METHOD

- Step 1:** Determine the cost table from the given problem. Examine whether total demand equals total supply then, go to step 2. Unless we introduce a dummy row/column having all its cost elements as zero and supply demand is the positive difference of supply and demand.
- Step 2:** Find the smallest cost in each row of the given cost matrix and then subtract the same from each cost of that row.
- Step 3:** In the reduced matrix obtained in step 2, locate the smallest cost of each column and then subtract the same from each cost of that column.
- Step 4:** For each row of the transportation table identify the smallest and the next - to - smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.
- Step 5:** Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference correspond to i^{th} row and let 0 be in the i^{th} row. Allocate the maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the $(i, j)^{\text{th}}$ cell and cross off either the i^{th} row or the j^{th} column in the usual manner.
- Step 6:** Again compute the column and row differences for the reduced transportation Table and go to step 5. Repeat the procedure until all the rim requirements are satisfied.

III.2. NUMERICAL EXAMPLE:

Consider Supplies and Demands are Dodecagonal Fuzzy Number.

1. Consider the following fuzzy transportation problem.

A company has three sources A_1, A_2, A_3 and three destinations B_1, B_2, B_3 , the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination C_{ij} .

$$\text{Where, } C_{ij} = \begin{bmatrix} (2,3,0,6,8,9,1,4,5,10,11,12) & (5,8,7,6,9,10,4,11,13,17,15,1) & (9,4,2,5,6,8,10,12,3,2,11,13) \\ (9,3,7,5,6,8,0,7,13,19,7,15) & (1,3,5,7,12,15,2,17,10,5,12,4) & (8,9,5,4,15,2,4,6,1,19,20,8) \\ (0,5,8,3,4,10,20,7,6,11,13,16) & (7,9,5,2,4,16,19,12,6,2,4,6) & (1,7,8,2,0,4,3,6,5,9,3,2) \end{bmatrix}$$

And the fuzzy availability of the supply are (8,8,3,6,10,3,9,2,5,1,4,6),(3,8,6,7,4,2,1,8,6,5,4,10),(3,5,6,10,12,15,20,24,7,3,4,8) and the fuzzy availability of the demand are (2,3,6,10,11,14,3,1,4,5,7,12),(10,12,14,18,8,6,2,4,6,9,3,16),(2,1,3,4,7,3,10,13,14,11,4,5) respectively.

Table 1

Destination Source	B_1	B_2	B_3	Supply
A_1	(2,3,0,6,8,9,1,4,5,10,11,12)	(5,8,7,6,9,10,4,11,13,17,15,1)	(9,4,2,5,6,8,10,12,3,2,11,13)	(8,8,3,6,10,3,9,2,5,1,4,6)
A_2	(9,3,7,5,6,8,0,7,13,19,7,15)	(1,3,5,7,12,15,2,17,10,5,12,4)	(8,9,5,4,15,2,4,6,1,19,20,8)	(3,8,6,7,4,2,1,8,6,5,4,10)
A_3	(0,5,8,3,4,10,20,7,6,11,13,16)	(7,9,5,2,4,16,19,12,6,2,4,6)	(1,7,8,2,0,4,3,6,5,9,3,2)	(3,5,6,10,12,15,20,24,7,3,4,8)
Demand	(2,3,6,10,11,14,3,1,4,5,7,12)	(10,12,14,18,8,6,2,4,6,9,3,16)	(2,1,3,4,7,3,10,13,14,11,4,5)	

The above fuzzy transportation problem is converted into a crisp value by using Robust Ranking method.

$$R(A_G) = \int_0^1 0.5\{(a_2 - a_1)\alpha + a_1, a_4 - (a_4 - a_3)\alpha, (a_6 - a_5)\alpha + a_5, a_8 - (a_8 - a_7)\alpha, (a_{10} - a_9)\alpha + a_9, a_{12} - (a_{12} - a_{11})\alpha\} d\alpha$$

The α - Cut of the fuzzy (2,3,0,6,8,9,1,4,5,10,11,12) number $R(a_{1,1})$ is

$$\begin{aligned} &= \int_0^1 0.5\{(3 - 2)\alpha + 2, 6 - (6 - 0)\alpha, (9 - 8)\alpha + 8, 4 - (4 - 1)\alpha, (10 - 5)\alpha + 5, 12 - (12 - 11)\alpha\} d\alpha \\ &= \int_0^1 0.5\{1\alpha + 2, 6 - 6\alpha, \alpha + 8, 4 - 3\alpha, 5\alpha + 5, 12 - \alpha\} d\alpha \\ &= \int_0^1 0.5\{-3\alpha + 37\} d\alpha \\ &= 0.5(35.5) = 17.75 \end{aligned}$$

Similarly, we get $R(a_{1,2}) = 26$, $R(a_{1,3}) = 21.25$, $R(a_{1,4}) = 14$, $R(a_{2,1}) = 24.25$, $R(a_{2,2}) = 22.75$, $R(a_{2,3}) = 25.25$, $R(a_{2,4}) = 12$, $R(a_{3,1}) = 25.75$, $R(a_{3,2}) = 23$, $R(a_{3,3}) = 12.5$, $R(a_{3,4}) = 27.5$, $R(a_{4,1}) = 19.5$

$R(a_{4,2}) = 27$, $R(a_{4,3}) = 34.5$.

Table 2 Crisp value

	B_1	B_2	B_3	Supply
A_1	17.75	26	21.25	14
A_2	24.25	22.75	25.25	12
A_3	25.75	23	12.5	27.5
Demand	19.5	27	34.5	

Total supply = 53.5, Total Demand = 81

The problem is unbalanced transportation problem. We have to convert it to balanced one by adding a Supply with zero cost. We need to add Supply 27.5 with zero cost.

The given problem becomes

Table 3 Convert balanced Table

	B_1	B_2	B_3	Supply
A_1	17.75	26	21.25	14
A_2	24.25	22.75	25.25	12
A_3	25.75	23	12.5	27.5
A_4	0	0	0	27.5
Demand	19.5	27	34.5	81

By using the above algorithm for solving the transportation problem we get the following allocations

Step 1: Determine the cost table from the given problem. Here total demand equals total supply, go to step 2.

Step 2: Locating the smallest element in each row of the given cost matrix and then subtracting the same element from each element of that row.

Table 4 MAM-First Allotment

	B_1	B_2	B_3	Supply
A_1	0	8.25	3.5	14
A_2	1.5	0	2.5	12
A_3	13.25	10.5	0	27.5
A_4	0	0	0	27.5
Demand	19.5	27	34.5	81

Step 3: In the reduced matrix obtained in step 2, locating the smallest element of each column and then Subtracting the same from each element of that column.

Table 5 MAM-Second Allotment

	B_1	B_2	B_3	Supply
A_1	0	8.25	3.5	14
A_2	1.5	0	2.5	12
A_3	13.25	10.5	0	27.5
A_4	0	0	0	27.5
Demand	19.5	27	34.5	81

Step 4: For each row of the transportation table identifying the smallest and the next - to -smallest costs. Determining the difference between them for each row in the transportation table. Displaying them along the side of the transportation table by enclosing them in parenthesis against the respective rows of the transportation table. Similarly computing the differences for each column of the transportation table.

Table 6 MAM-Third Allotment

	B_1	B_2	B_3	Supply	Penalty
A_1	0	8.25	3.5	14	3.5
A_2	1.5	0	2.5	12	1.5
A_3	13.25	10.5	0	27.5	10.5
A_4	0	0	0	27.5	0
Demand	19.5	27	34.5	81	0
Penalty	0	0	0		

Step 5: Identifying the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tiebreaking choice. Let the greatest difference correspond to i^{th} row and let 0 be in the i^{th} row. Allocating the maximum feasible amount x_{ij} , such that $x_{ij} = \min(a_i, b_j)$ in the (i, j) th cell and cross off either the i^{th} row or the j^{th} column in the usual manner.

	B_1	B_2	B_3	Supply	Penalty
A_1	0	8.25	3.5	14	3.5
A_2	1.5	0	2.5	12	1.5
A_3	13.25	10.5	0[27.5]	27.5[0]	10.5
A_4	0	0	0	27.5	0
Demand	19.5	27	34.5[7]	81	0
Penalty	0	0	0		

Step 6: Again compute the column and row differences for the reduced transportation Table and go to step 5. Repeat the procedure until all the rim requirements are satisfied.

Table 7 Reduced Table of MAM Method

	B_1	B_2	B_3	Supply
A_1	0[14]	8.25	3.5	14
A_2	1.5	0[12]	2.5	12
A_3	13.25	10.5	0[27.5]	27.5
A_4	0[5.5]	0[15]	0[7]	27.5
Demand	19.5	27	34.5[7]	81

	B_1	B_2	B_3	Supply
A_1	17.75[14]	26	21.25	14
A_2	24.25	22.75[12]	25.25	12
A_3	25.75	23	12.5[27.5]	27.5

A_4	0[5.5]	0[15]	0[7]	27.5
Demand	19.5	27	34.5	

The above table satisfies the rim conditions with $(m+n-1)$ non negative allocations at independent positions. The transportation cost according to the proposed method is:

$$\text{Total cost} = (17.75 * 14) + (22.75 * 12) + (12.5 * 27.5) + (0 * 5.5) + (0 * 15) + (0 * 7) = 865.25$$

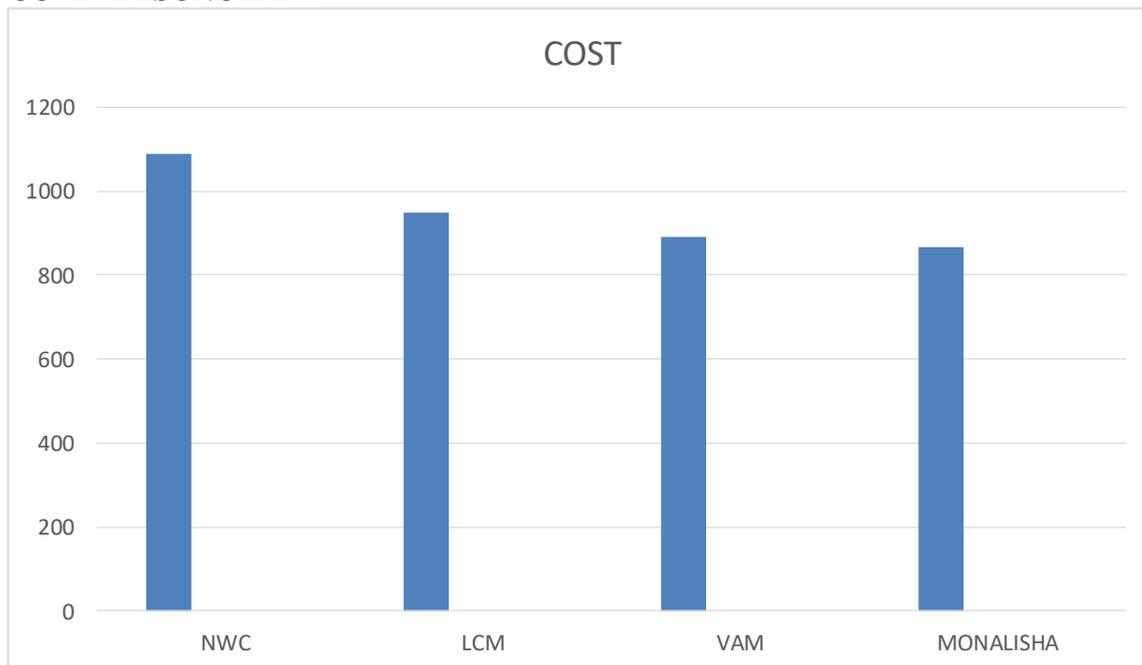
Total minimum cost will be Rs.865.25.

In order to show the efficiency of the proposed method, the same problem is solved with various methods like North West corner (NWC), Least cost method (LCM) and Vogel's Approximation method (VAM). We get the following results after solving the problem.

COMPARISON WITH EXISTING METHOD

NORTH WEST CORNER METHOD	LEAST COST METHOD	VOGAL'S APPROXIMATION METHOD	MONALISHA APPROXIMATION METHOD
1088.75	947.5	890.25	865.25

COMPARISON CHART



IV. CONCLUSIONS

In this paper, we have shown that the maximal profit obtained using Dodecagonal fuzzy number gives more profit in comparison with North -West corner method, Least cost method, Vogel's Approximation method. It

is concluded that Dodecagonal Fuzzy Transportation method proves to be minimum cost of Transportation by using Monalisha's Approximation Method.

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